(DM01)

M.Sc.(Previous) DEGREE EXAMINATION, DECEMBER – 2015

(First Year)

MATHEMATICS

Paper - I : Algebra

Time : 3 Hours **Maximum Marks: 70**

Answer any five of the following

All questions carry equal Marks

1) a) Let ϕ be a homormorphism of G onto $\overline{G}$ with Kernel. Then prove that $G/K \cong \overline{G}$.

b) State and prove Cauchy's theorem for abelian groups.

2) a) If $O(G) = P^n$, where P is prime number, then show that $Z(G) \neq (e)$.

b) Prove that the number of conjugate class in S_n is $P(n)$, the number of partitions of n.

3) a) State and prove second part of Sylow's theorem.

b) Prove that the number of nonisomorphic abelian groups of order P^n, P*a* prime, equals the number of partitions of n.

4) a) Prove that a finite integral domain is a field.

b) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then prove that R is field.

5) a) State and prove the Eisenstein criterion theorem.

b) If $a \in R$ is an irreducible element and $a|bc$, then show that $a|b$ or $a|c$.

6) a) If L is a finite extension of K and if K is a finite extension of F, then show that L is a finite extension of F. Moreover $[L:F] = [L:K]\,[K:F]$.

b) Prove that the number e is transcendental.

7) a) Prove that S_n is not solvable for $n \geq 5$.

b) If $P(x) \in F(x)$ is solvable by radicals over F, then prove that the Galois group over F of $P(x)$ is a solvable group.

8) State and Prove the fundamental theorem of Galois theory.

9) State and prove Schreier's theorem.

10) Prove that every distributive lattice which more than one element can be represented as a subdirect union of two element chains.

(DM02)

M.Sc.(Previous) DEGREE EXAMINATION, DECEMBER – 2015

First Year

MATHEMATICS

Paper - II : Analysis

Time : 3 Hours **Maximum Marks: 70**

Answer any five of the following

All questions carry equal Marks

1) a) Prove that every infinite subset of a countable set A is countable.

b) Prove that compact subsets of metric spaces are closed.

2) a) Prove that every k-cell is compact.

b) Let P be a nonempty perfect set in R^k. Then show that P is uncountable.

3) a) Show that the product of two convergent series need not converge and may actually diverge.

b) Suppose $\{S_n\}$ is monotonic. Then show that $\{S_n\}$ converges if and only if it is bounded.

4) a) Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Then show that $f(x)$ is compact.

b) Let f be a continuous mapping of a compact metric space X into a metric space Y. Then show that f is uniformly continuous on X.

5) a) Let f be monotonic on (a, b). Then show that the set of points of (a, b) at which f is discontinuous is at most countable.

b) If f is continuous on $[a, b]$, then show that $f \in R(\alpha)$ on $[a, b]$.

6) a) Suppose $f \in R(\alpha)$ on $[a, b]$, $m \leq f \leq M$, ϕ is continuous on $[m, M]$, and $h(x) = \phi(f(x))$ on $[a, b]$. Then show that $h \in R(\alpha)$ on $[a, b]$.

b) Assume α increases monotonically and $\alpha' \in R$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Then show that $f \in R(\alpha)$ if and only if $f\alpha' \in R$. In that case $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)\,dx$.

7) a) Prove that the sequence of functions $\{f_n\}$, defined on E, converges uniformly on E if and only if for every $\varepsilon > 0$ there exists on integer N such that $m \geq N$, $n \geq N$, $x \in E$ implies $|f_n(x) - f_m(x)| \leq \varepsilon$.

b) Prove that there exists a real continuous function on the real line which is now differentiable.

8) State and Prove Weierstrass approximation theorem.

9) a) Let f and g be measurable real valued functions defined on X, let F be real and continuous on R^2, and put $h(x) = F(f(x), g(x))$, $(x \in X)$.

Then show that h is measurable. In particular, $f + g$ and fg are measurable.

b) State and prove Fatou's theorem.

10) a) If $f \in \mathcal{L}(\mu)$ on E, then show that $|f| \in \mathcal{L}(\mu)$ on E and $\left|\int_E f d\mu\right| \leq \int_E |f|\, d\mu$.

b) State and prove Lebesgue's dominated convergence theorem.

M.Sc.(Previous) DEGREE EXAMINATION, DECEMBER – 2015

First Year

MATHEMATICS

Paper - III: Complex Analysis and Special Functions and Partial Differential Equations

Time : 3 Hours **Maximum Marks: 70**

Answer Any five questions

Choosing atleast two from each section

All questions carry equal marks

SECTION-A

1) a) Prove that $(2n+1)x\ p_n(x) = (n+1)\ p_{n+1}(x) + n\ p_{n-1}(x)$.

b) Prove that $J_{5/2}(x) = \sqrt{\left(\frac{2}{\pi x}\right)}\left\{\frac{3-x^2}{x^2}\sin x - \frac{3}{x}\cos x\right\}$

2) a) Prove that $\int_{-1}^{1} x\ p_n p_m' dx$ either 0 or 2 or $\frac{2n}{2n+1}$.

b) Show that $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$.

3) a) Evaluate $\int x^3 J_3(x) dx$,

b) Derive the Rodrigue's formula.

4) a) Solve $(r + s - 6t) = y\cos x$.

b) Solve $(yz + 2x)\ dx + (zx - 2z)\ dy + (xy - 2y) dz = 0$.

5) a) Solve $py + qx = xyz^2(x^2 - y^2)$.

b) Find the complete integral of the equation $2(z + xp + yq) = yp^2$.

SECTION-B

6) a) If *f(z)* is an analytic function, Prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$.

b) Find the radius of convergence of $\sum_{n=1}^{\infty} n^n z^n, z \in c$.

7) a) i) State and prove the symmetry principle.

ii) Discuss the mapping properties of cos z and sin z .

b) If r is a piecewise smooth and $f:[a,b]\to c$ is continuous then prove that

$$\int_a^b f dr=\int_a^b f(t)r'(t)dt$$

8) a) State and prove Liouville's theorem. Deduce the fundamental theorem of algebra.

b) State and prove Cauchy's Integral formula.

9) a) State and prove Schwarz's lemma.

b) Prove that $\int_0^{2\pi}\frac{d\theta}{2-\sin\theta}=\frac{2\pi}{\sqrt{3}}$

10) a) Show that $\int_0^{\infty}\frac{x^{-a}}{1+x}dx=\frac{\pi}{\sin \pi a}$ if $0<a<1$ using residue theory.

b) State and prove Rouche's theorem.

◀▶◀▶◀▶

(DM 04)

M.Sc.(Previous) DEGREE EXAMINATION, DECEMBER – 2015

First Year

MATHEMATICS

Paper – IV : Theory of Ordinary Differential Equations

Time : 3 Hours **Maximum Marks: 70**

Answer Any five questions.

All questions carry equal marks.

1) a) Show that there exists n linearly independent solutions of L(y)=0. On I.

b) If $\phi_1, \phi_2,$ ------, ϕ_n are n solutions of L(y)=0 on an interval I, show that they are linearly independent there if and only if, $w(\phi_1, \phi_2, \text{------}, \phi_n)\,(x) \neq 0$ for all x in I.

2) a) Find all solutions of $x^2 y'' - 4xy' + (2 + x^2)y = x^2$ for $x>0$.

b) Find two linearly independent solutions of $y'' - 2xy' + 2\alpha y = 0$,where α is constant.

3) a) Compute the first four successive approximations $\phi_0, \phi_1, \phi_2, \phi_3$ for the equation $y' = 1 + xy$, $y(0) = 1$.

b) Let M,N be two real-valued functions which have continuous first partial derivatives on some rectangle $\mathrm{R} : |x - x_0| \leq a, |y - y_0| \leq b.$ Then show that the equation $\mathrm{M}(x, y) + \mathrm{N}(x, y)y' = 0$ is exact in R it ,and only if, $\frac{\partial \mathrm{M}}{\partial y} = \frac{\partial \mathrm{N}}{\partial x}$ in R.

4) a) Show that the function f given by $f(x, y) = y^{1/2}$ does not satisty a Lipscnitz condition on $\mathrm{R} : |x| \leq 1 \;, 0 \leq y \leq 1$.

b) Find the integrating factor and solve the equation $(e^y + xe^y)dx + xe^y dy = 0$.

5) a) Find the solution ϕ of $y'' = 1 + (y')^2$ which satisties $\phi(0) = 0, \phi'(0) = -1$.

b) State and prove Local existence theorem.

6) a) Find a solution ϕ of the system $y_1' = y_1, y_2' = y_1 + y_2$ which satisties $\phi(0) = (1, 2)$.

b) Consider the system $y_1' = 3y_1 + xy_3, y_2' = y_2 + x^3 y_3, y_3' = 2xy_1 - y_2 + e^x y_3$. Show that every initial value problem for this system has a unique solution which exists for all real x.

7) a) Find a function *z(x)* such that $z(x)[y'' + y] = \frac{d}{dx}[k(x)y'm(x)y]$.

b) Derive an adjoint equation for $Ly \equiv y' - Ay = 0$, where A is n×n matrix and obtain a condition for the operator L to be self adjoint.

8) a) Study the solutions of the Riccati equation $z' + z - e^z, z^2 - e^{-z} = 0$.

b) Construct Green's function for the problem $u'' = 0, u(0) = 0, u(1) = 0$.

9) a) State and prove sturm- Picone theorem.

b) Prove that *if* $\int_1^\infty \left[xp(x) - \frac{1}{4x} \right] dx = +\infty$, the solutions of $y'' + p(x)y = 0$ are oscillatory on $(1, \infty)$.

10) a) State and prove sturm comparison theorem.

b) Prove that between every pair of consecutive zeros of sin x there is one zero of sin x + cos x.

◀▶◀▶◀▶